

Pablo Araya (U. de Chile): Non-existence of the level sets of the GFF in higher dimensions

Abstract:

The Gaussian Free Field (GFF) can be viewed as a generalization of Brownian motion when the time domain is replaced by a d -dimensional domain. One natural way to study its geometry is through its exit sets, which in $d=1$ are directly related to Brownian motion exit times. In $d=2$, recent work has shown that these sets can still be defined, although only under strong restrictions that reflect the fact that the GFF is no longer a function but a random distribution. In dimensions three and higher, however, not that much is known, and even the existence of such exit sets is unclear.

In this talk, I will present an approach to this problem based on a new notion for random sets, which we call explorable sets. Roughly speaking, these are sets that can be discovered in an adapted way. I will first explain this concept in an abstract setting and then describe how it applies to the GFF. In particular, I will show that a natural observable of the exit sets has the same law as the corresponding Brownian motion exit times, in any dimension. Finally, I will tell how this new framework can be used to study the existence of exit sets in higher dimensions, and how it leads to non-existence results in sufficiently high dimensions.

Based on joint work with Avelio Sepúlveda and Pablo Zúñiga.