

Damian Cid (U. de Chile): The pseudo-inverse of the gradient via uniform spanning trees

Abstract:

We study the discrete gradient operator on a finite rooted graph, mapping vertex functions (0-forms) to functions on oriented edges (1-forms) via $df(e) = f(e^+) - f(e^-)$. On a rooted tree, acyclicity implies that the gradient is invertible after fixing the function value at the root, yielding a well-defined primitive along the tree. For a general rooted graph, each spanning tree defines a right inverse of the gradient.

Our main result shows that averaging these primitives with respect to the uniform spanning tree measure yields the pseudo-inverse of the gradient operator. The proof relies on Wilson's algorithm and on the observation that loop integrals cancel in expectation, allowing the expected value of the primitive at a vertex to be represented as the average of the integral along a random walk path.