Convergence of Elo rating systems

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Abstract

The Elo rating system is a popular and widely adopted method for measuring the relative skill levels of players or teams in various sports and competitions, including chess, Go, football, online games, etc. The system assigns players numerical ratings and dynamically updates them based on game results and a model parameter K, which determines the sensitivity of rating changes. Assuming random games, this leads to a Markov chain for the evolution of the ratings of the N players in the league. Despite its widespread use, little is known about the long-term behavior of this process. Aiming to fill this gap, in this talk we show that the process converges to its unique equilibrium distribution in an almost-sure sense, as well as in the Wasserstein-2 distance at an exponential rate. Moreover, we show important properties of the stationary distribution, such as the finiteness of an exponential moment, full support, and convergence to the players' true skills as K decreases, at a rate of $\sqrt{K}$. We also provide Monte Carlo simulations that illustrate some of these properties and offer new insights.